CLASA A XII-A
BAREM

(1) $xy = x(y^3)^5[(y^{-1})^2]^7$ (2p) $= (y^3)^5 x[(y^{-1})^2]^7$ (2p)

$= (y^3)^5 \cdot [(y^{-1})^2]^7 x$ (2p) $= yx$. (1p)

(2) $G = \operatorname{Im} f \Rightarrow G = (1, \infty)$ (2p)

$f(xy) = f(x) \circ f(y)$ (2p)

$2xy+1 = (2x+1) \circ (2y+1)$ (1p) $\quad 2x+1 = u$, $2y+1 = v$ (1p)

$u \circ v = 2 \cdot \frac{u-1}{2} \cdot \frac{v-1}{2} + 1$ (1p)

(3) $t = \frac{\pi}{2} - x \Rightarrow I = \int_0^{\pi/2} \frac{\sin t + \sin^2 t}{1 + \cos t + \sin t} dt$ (2p)

$2I = \int_0^{\pi/2} \frac{(\cos x + \cos^2 x) + (\sin x + \sin^2 x)}{1 + \sin x + \cos x} dx$ (2p)

$2I = \int_0^{\pi/2} dx$ (2p) $\quad I = \pi/4$ (1p)

(4) $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx = bf(b) - af(a)$

(fără a demonstra) (2p)

$f(x) = \sqrt{x^2+1} \Rightarrow \underbrace{\int_0^1 \sqrt{x^2+1}\,dx}_{I} + \underbrace{\int_1^{\sqrt{2}} \sqrt{x^2-1}\,dx}_{J} = \sqrt{2}$ (2p)

$\sqrt{I \cdot J} < \frac{I+J}{2} = \frac{\sqrt{2}}{2}$ (2p) $\Rightarrow I \cdot J < \frac{1}{2}$ (1p)