Barem de corectare OLM Clasa a IX-a, 2014

1. P(1): $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\triangleright 1 \Leftrightarrow \frac{13}{12}\triangleright 1$, inegalitate adevarata ....**(2p)**

$k\geq 1$, P(k) adevarată$\Rightarrow$ P(k+1): $\frac{1}{k+2}+\frac{1}{k+3}+...+\frac{1}{3k+1}+\frac{1}{3k+2}+\frac{1}{3k+3}+\frac{1}{3k+4}\triangleright 1$..**(2p)**

Este suficient să arătăm că: $\frac{1}{3k+2}+\frac{1}{3k+3}+\frac{1}{3k+4}-\frac{1}{k+1}>1$ ....**(1p)**

Finalizare..**(2p)**

2. $x=[x]+\{x\}\Rightarrow[x]+[2x]+[3x]=6x-1$ ....**(2p)**

$6x-1=k\in Z\Rightarrow x=\frac{k+1}{6}$ ....**(2p)**

Ecuatia devine $\left[\frac{k+1}{6}\right]+\left[\frac{k+1}{3}\right]+\left[\frac{k+1}{2}\right]=k$ ....**(1p)**

Se considera k de forma 6p, 6p+1, 6p+2, ..., 6p+5, $p\in\mathbf{Z}$ și se obține mulțimea soluțiilor $S=\left\{\frac{6p+1}{6},\frac{6p+2}{6},\frac{6p+3}{6},\frac{6p+4}{6}\middle| p\in Z\right\}$ ....**(2p)**

3. Pentru n=1 avem $a_1=\frac{(a_1+1)^2}{4}\Leftrightarrow(a_1-1)^2=0\Leftrightarrow a_1=1$ ....**(1p)**

Pentru n=2 avem $a_1+4a_2=(a_2+1)^2\Leftrightarrow a_2(a_2-2)=0\Rightarrow a_2=2$ ....**(1p)**

Demonstrăm prin inducție matematică $a_n=n,(\forall)n\in N^*$.

$k\geq 2$, $a_k=k$ și P(k+1): $1^2a_1+2^2a_2+...+k^2a_k+(k+1)^2a_{k+1}=\frac{(k+1)^2(a_{k+1}+1)^2}{4}\Leftrightarrow$ ....**(1p)**

$\Leftrightarrow\frac{k^2(a_k+1)^2}{4}+(k+1)^2a_{k+1}=\frac{(k+1)^2(a_{k+1}+1)^2}{4}\Leftrightarrow(a_{k+1}-k-1)(a_{k+1}+k-1)=0$ ....**(2p)**

$\Rightarrow a_{k+1}=k+1$, de unde $a_n=n,(\forall)n\in N^*$ ....**(2p)**

4. a) Figura corectă ....**(1p)**

$\overrightarrow{DM}+\overrightarrow{EN}=\frac{\overrightarrow{DA}+\overrightarrow{DB}}{2}+\frac{\overrightarrow{EA}+\overrightarrow{EC}}{2}=-\frac{\overrightarrow{AD}+\overrightarrow{AE}}{2}$ ....**(1p)**

$\overrightarrow{AD}+\overrightarrow{AE}=2\overrightarrow{AQ}=-2\overrightarrow{QA}$ ....**(1p)**

$\overrightarrow{DM}+\overrightarrow{EN}=\overrightarrow{QA}\Rightarrow r=1$ ....**(1p)**

b) I $MN\cap AQ=\{P\}\Rightarrow P$ este mijlocul lui [MN] ....**(1p)**

$\frac{PG_1}{G_1D}=\frac{1}{2}=\frac{PG_2}{G_2E}\Rightarrow G_1G_2\|DE\Rightarrow G_1G_2\|BC$ ....**(2p)**

II. Fie P un punct oarecare$\Rightarrow\overrightarrow{PM}+\overrightarrow{PN}+\overrightarrow{PD}=3\overrightarrow{PG_1}$, $\overrightarrow{PM}+\overrightarrow{PN}+\overrightarrow{PE}=3\overrightarrow{PG_2}$ ....**(1p)**

$3\overrightarrow{G_2G_1}=3\left(\overrightarrow{PG_1}-\overrightarrow{PG_2}\right)=\overrightarrow{PD}-\overrightarrow{PE}=\overrightarrow{ED}\Rightarrow G_1G_2\|BC$ ....**(2p)**