Olimpiada de Matematică – Etapa Locală
Maramureş – 29 februarie 2020
Clasa a VII - a
Barem de corectare şi notare

1. a) $\frac{1}{2^{n-1}}-\frac{1}{2^n}=\frac{2}{2^n}-\frac{1}{2^n}=\frac{1}{2^n}\quad \forall n\in \mathbb{N}^*$. ……………………………………………2p

b) $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\ldots+\frac{1}{3\cdot 2^n}=\frac{1}{3}\left(2+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^n}\right)$ ……………...…………….1p

$=\frac{1}{3}\left(3+\frac{1}{2^0}-\frac{1}{2}+\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^2}-\frac{1}{2^3}+\ldots+\frac{1}{2^{n-1}}-\frac{1}{2^n}\right)$...2p

$=\frac{1}{3}\left(3+\frac{1}{2^0}-\frac{1}{2^n}\right)$

……………………..…..………1p

$=\frac{1}{3}\left(4-\frac{1}{2^n}\right)=\frac{4}{3}-\frac{1}{3\cdot 2^n}<\frac{4}{3}$ …………………..……1p

2. a) $a=\frac{\sqrt{2}}{\sqrt{2}}-\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{6}}-\frac{\sqrt{2}}{\sqrt{6}}+\ldots+\frac{\sqrt{100}}{\sqrt{9900}}-\frac{\sqrt{99}}{\sqrt{9900}}$
…………………..……………..1p

$=1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{99}}-\frac{1}{\sqrt{100}}=1-\frac{1}{10}=\frac{9}{10}$…………..……… …………1p

$b=\left(\frac{\sqrt{3}+\sqrt{2}}{2}\right)^1-\frac{\left|\sqrt{2}-\sqrt{3}\right|}{2}=\frac{\sqrt{3}+\sqrt{2}}{2}-\frac{\sqrt{3}-\sqrt{2}}{2}=\frac{\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2}}{2}=\frac{2\sqrt{2}}{2}=\sqrt{2}$ …..….1p

n = $\sqrt{30\cdot\frac{9}{10}-\left(\sqrt{2}\right)^2}=\sqrt{27-2}=\sqrt{25}=5\in N$ ……….…………………..1p

b) Obsevăm că $(a,b)=1,\ (a,c)=1,\ (c,d)=1.$ ……………………………….1p

Rezultă că $[a,b]+[a,c]=(n+3)(2n+7)+(n+3)(2b+5)=4(n+3)^2$……1p
Deci $A=\sqrt{\frac{4\left(n+3^2\right)}{1}}=2(n+3)$. ………………………………………..………1p

3. $[OM]$ linie mijlocie în $\Delta DOC \Rightarrow OM \parallel AD \Rightarrow OM\perp DC$ 2p
$A_{\Delta OMC}=18\,cm^2$……………………………………………………………..……….1p
În $\Delta\, ADC$, $[AM]$ şi $[DO]$ sunt mediane cu $AM\cap DO=\{N\}$. ……………………..…1p

$A_{\Delta MON}=\frac{NO\cdot d(M,NO)}{2}=\frac{\frac{1}{3}DO\cdot d(M,DO)}{2}=\frac{1}{3}A_{\Delta MOD}=6\,cm^2$……………..………..2p

$A_{\Delta CONM} = A_{\Delta OMC} + A_{\Delta MON} = 24\,cm^2$. 1p

4. Fie N simetricul punctului C față de punctul 1p
$[BE] \equiv [BN]$ din $BECD$ paralelogram şi $[BE] \equiv [DC]$ ……………………………..1p
$\sphericalangle FBE \equiv \sphericalangle FBN$ ………………………………………………………………… 1p
$\Delta FBE \equiv \Delta FBC$ (L.U.L.) ……………………………………….…………………..…1p
$[FE] \equiv [EN] = [FN]$, deci ΔEFN este echilateral …………….………………..……1p
$m(\sphericalangle BEN) = 45^o$ ……………………………………………………………..……..1p
$m(\sphericalangle FEB) = 60^o - 45^o = 15^o$………………………..…………………………….1p