BAREM DE CORECTARE

OLIMPIADA DE MATEMATICĂ

ETAPA LOCALĂ - 17 februarie 2018

Clasa a XI - a

SUBIECTUL I (7p)

$det(xI_3 - A) = (x-1)^3 \Rightarrow x = 1$ .... $(2p)$

$A = I_3 + B$ unde $B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow B^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ și $B^k = O_3$, $(\forall) k \geq 3$ .... $(2p)$

$I_3 \cdot B = B \cdot I_3 (= B)$ .... $(1p)$

$A^n = (I_3 + B)^n = I_3 + C_n^1 B + C_n^2 B^2 \Rightarrow$ .... $(1p)$

$A^n = \begin{pmatrix} 1 & 0 & n \\ n & 1 & \frac{n(n-1)}{2} \\ 0 & 0 & 1 \end{pmatrix}$ .... $(1p)$

SUBIECTUL II (7p)

$\frac{1}{2}a \leq f(1) \leq a$; $\frac{1}{2}a^2 \leq f(2) \leq a^2$; .... ; $\frac{1}{2}a^n \leq f(n) \leq a^n$ .... $(2p)$

Prin sumare $\Rightarrow \frac{1}{2} \cdot \frac{a^{n+1} - a}{a-1} \leq f(1) + f(2) + ... + f(n) \leq \frac{a^{n+1} - a}{a-1}$ .... $(1p)$

$\Rightarrow \sqrt[n]{\frac{1}{2} \cdot \frac{a^{n+1} - a}{a-1}} \leq a_n \leq \sqrt[n]{\frac{a^{n+1} - a}{a-1}}$ .... $(1p)$

$\lim_{n\to\infty} \sqrt[n]{\frac{1}{2} \cdot \frac{a^{n+1} - a}{a-1}} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{2}} \lim_{n\to\infty} \sqrt[n]{\frac{a^{n+1} - a}{a-1}} = 1 \cdot a = a$ .... $(2p)$

$\lim_{n\to\infty} a_n = a$ .... $(1p)$

SUBIECTUL III (7p)

a) $det\left(X^2\right)=det\,A \Rightarrow det\,A=\left(det\,X\right)^2 \geq 0 \Rightarrow$ pentru $det\,A<0$ nu avem soluții .... $(1p)$

b) $det\,A>0 \Rightarrow det\,X=\pm\sqrt{det\,A} \Rightarrow X^2-trX\cdot X\pm\sqrt{det\,A}\cdot I_2=O_2 \Rightarrow trX\cdot X=A\pm\sqrt{det\,A}\cdot I_2\,(*)$ .... $(2p)$

$\Rightarrow\left(trX\right)^2=tr\left(A\pm\sqrt{det\,A}\cdot I_2\right)\Rightarrow\left(trX\right)^2=trA\pm2\sqrt{det\,A}$ .... $(1p)$

dacă $trA>2\sqrt{det\,A}\Rightarrow trA\pm2\sqrt{det\,A}>0\Rightarrow trX=\pm\sqrt{trA\pm2\sqrt{det\,A}}\overset{(*)}{\Rightarrow}$ patru soluții.................................... $(1p)$

c) dacă $0<trA<2\sqrt{det\,A}\Rightarrow trA-2\sqrt{det\,A}<0$ și cum $\left(trX\right)^2\geq0\Rightarrow$ .... $(1p)$

$\left(trX\right)^2=trA+2\sqrt{det\,A}\Rightarrow trX=\pm\sqrt{trA+2\sqrt{det\,A}}\overset{(*)}{\Rightarrow}$ două soluții.. $(1p)$

SUBIECTUL IV (7p)

$a_n=\frac{1}{2}\sum_{k=1}^{n}\frac{2k}{\left(k^2-k+1\right)\left(k^2+k+1\right)}=\frac{1}{2}\left(1-\frac{1}{n^2+n+1}\right)$ .... $(2p)$

$\Rightarrow 1-2a_k=\frac{1}{k^2+k+1}$ .... $(1p)$

$\Rightarrow\sum_{k=1}^{n}arctg\left(1-2a_k\right)=\sum_{k=1}^{n}arctg\frac{1}{k^2+k+1}=\sum_{k=1}^{n}arctg\frac{k+1-k}{k^2+k+1}=\sum_{k=1}^{n}\left(arctg\left(k+1\right)-arctgk\right)$ .... $(2p)$

$\Rightarrow\lim_{n\to\infty}\sum_{k=1}^{n}arctg\left(1-2a_k\right)=\lim_{n\to\infty}\left(arctg\left(n+1\right)-arctg1\right)=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$ .... $(2p)$