CLASA A XI A
BAREM

① $A^3 = -I_2$ (3p) $A^{2025} = (A^3)^{675}$ (2p)
$= (-I_2)^{675}$ (1p) $= -I_2$ (1p)

② a) $\lim_{n\to\infty} \sqrt[n]{(2n)!} = \lim_{n\to\infty} \frac{(2n+2)!}{2n!}$ (2p)
$= \lim_{n\to\infty} (2n+1)(2n+2) = \infty$ (1p)

b) $1 < \sqrt[n^3]{(2n)!} < \sqrt[n^3]{(2n)\cdot(2n)\cdots(2n)}$ (1p)
$= (2n)^{\frac{2}{n^2}} < (2n)^{\frac{2}{n}}$ (1p) $\longrightarrow 1$ (2p)
$\Rightarrow \lim_{n\to\infty} \sqrt[n^3]{(2n)!} = 1.$

③ $f(x) \le f\left(\frac{x}{2^n}\right)$ și $f(x) \ge f\left(\frac{x}{3^n}\right)$ (2p)
pentru $n \to \infty$: $f(x) \le l$ și $f(x) \ge l$ (2p)
$f(x) = l,\ x \neq 0$ (2p) $f(0) = \alpha$ oarecare (1p)

④ $a_n < 3$ (inductie) (1p)
$a_{2n} = \sqrt{6 + a_{2n-1}} = \sqrt{6 + \sqrt[3]{24 + a_{2n-2}}}$ (1p)
$a_0 < a_2 \Rightarrow (a_{2n})$ crescc (ind.) (1p)
$a_{2n+1} = \sqrt[3]{24 + a_{2n}} = \sqrt[3]{24 + \sqrt{6 + a_{2n-1}}}$ (1p)
$a_1 < a_3 \Rightarrow (a_{2n+1})$ crescc (ind) (1p)
$a_{2n} \longrightarrow 3$; $a_{2n+1} \longrightarrow 3 \Rightarrow a_n \longrightarrow 3$ (2p)