BAREM DE CORECTARE SI NOTARE
Clasa a XI a - Matematică-informatică
Olimpiada de matematică - Faza locală 17.02.2020

- *Nu se acordă puncte din oficiu.*
- *Pentru orice soluție corecta, diferită de cea din barem, se acordă punctajul corespunzător.*
- *Fiecare exercițiu este punctat de la 0 la 7.*

SUBIECTUL I

a)

$$A^2=\begin{pmatrix}1&0&0\\2&1&0\\2a+3&6&1\end{pmatrix}$$ ***2p***

$${}^t\left(A^2\right)=\begin{pmatrix}1&2&4043\\0&1&6\\0&0&1\end{pmatrix}\Leftrightarrow a=2020$$ ***2p***

b)

$$A(a)=\begin{pmatrix}1&0&0\\1&1&0\\a&3&1\end{pmatrix}=I_3+\underbrace{\begin{pmatrix}0&0&0\\1&0&0\\a&3&0\end{pmatrix}}_{B}$$ ***1p***

Calculul $B^2=\begin{pmatrix}0&0&0\\0&0&0\\3&0&0\end{pmatrix}$ si $B^3=O_3$ ***1p***

$$A^{2020}=\begin{pmatrix}1&0&0\\2020&1&0\\3\cdot1010\cdot2019+2020a&6060&1\end{pmatrix}$$ ***1p***

SUBIECTUL II

a)

$$\Delta_1=\begin{vmatrix}1&1&1\\a&b&c\\a^2&b^2&c^2\end{vmatrix}\overset{\substack{C_2-C_1\\C_3-C_1}}{=}\begin{vmatrix}1&0&0\\a&b-a&c-a\\a^2&b^2-a^2&c^2-a^2\end{vmatrix}=(b-a)(c-a)\begin{vmatrix}1&1\\b+a&c+a\end{vmatrix}$$ ***2p***

$$\Delta_1=(a-b)(b-c)(c-a)$$ ***1p***

b)

$$\Delta_2=\begin{vmatrix}1&1&1\\a&b&c\\a^2+2a+3&b^2+2b+3&c^2+2c+3\end{vmatrix}=\begin{vmatrix}1&1&1\\a&b&c\\a^2&b^2&c^2\end{vmatrix}+\begin{vmatrix}1&1&1\\a&b&c\\2a&2b&2c\end{vmatrix}+\begin{vmatrix}1&1&1\\a&b&c\\3&3&3\end{vmatrix}=$$ ***1p***

$$\begin{vmatrix}1&1&1\\a&b&c\\a^2&b^2&c^2\end{vmatrix}=\Delta_1$$ ***1p***

c)

Fie $A(a,f(a)),B(b,f(b)),C(c,f(c))\Leftrightarrow A_{\Delta ABC}=\frac{1}{2}\left\|\begin{matrix}a&f(a)&1\\b&f(b)&1\\c&f(c)&1\end{matrix}\right\|=\frac{1}{2}\left|(a-b)(b-c)(c-a)\right|$ ***1p***

$a,b,c\in\mathbb{N}$, deci cel putin 2 au aceeasi paritate $\Rightarrow$ diferenta lor este nr par => $A\in\mathbb{N}$ ***1p***

SUBIECTUL III

a) $a_n=\dfrac{1+2+...+n}{1^2+2^2+...+n^2}=\dfrac{3}{2n+1}$ ***1p***

$$\lim_{n\to\infty}\left(\frac{2n}{3}\cdot a_n\right)^{n-1}=\lim_{n\to\infty}\left(\frac{2n}{2n+1}\right)^{n-1}=\lim_{n\to\infty}\left(1+\frac{-1}{2n+1}\right)^{n-1}=\lim_{n\to\infty}\left[\left(1+\frac{-1}{2n+1}\right)^{\frac{2n+1}{-1}}\right]^{\frac{-1}{2n+1}\cdot(n-1)}=e^{-\frac{1}{2}}$$ ***3p***

b) ***2p***

$$\lim_{n\to\infty}\left(\frac{tg\left(p\cdot a_n\right)}{\ln\left(1+(1+p)\cdot a_n\right)}\right)=\lim_{n\to\infty}\left(\frac{\frac{tg\left(p\cdot a_n\right)}{p\cdot a_n}\cdot p\cdot a_n}{\frac{\ln\left(1+(1+p)\cdot a_n\right)}{(1+p)\cdot a_n}\cdot(1+p)\cdot a_n}\right)=\lim_{n\to\infty}\left(\frac{p\cdot a_n}{(1+p)\cdot a_n}\right)=\frac{p}{p+1}=\frac{2020}{2021}$$

$\Leftrightarrow p=2020$.

SUBIECTUL IV

Trebuie ca $\lim\limits_{x\to\infty}f(x)=1$ ***1p***

$$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}x\left(\frac{1-\frac{4}{x}+\frac{3}{x^2}}{\sqrt{a+\frac{b}{x}+\frac{2020}{x^2}}}-1\right)=\frac{1}{\sqrt{a}}\lim_{x\to\infty}x\left(1-\frac{4}{x}+\frac{3}{x^2}-\sqrt{a+\frac{b}{x}+\frac{2020}{x^2}}\right)=$$ ***1p***

$$\frac{1}{\sqrt{a}}\lim_{x\to\infty}\left[x\left(1-\sqrt{a+\frac{b}{x}+\frac{2020}{x^2}}\right)-4+\frac{3}{x}\right]=\frac{1}{\sqrt{a}}\left[-4+\lim_{x\to\infty}x\left(1-\sqrt{a+\frac{b}{x}+\frac{2020}{x^2}}\right)\right]$$ ***1p***

Pentru ca $\lim\limits_{x\to\infty}f(x)=1\Leftrightarrow 1-\sqrt{a}=0\Leftrightarrow a=1$ ***1p***

$$\lim_{x\to\infty}f(x)=\frac{1}{\sqrt{1}}\left(-4+\lim_{x\to\infty}x\left(1-\sqrt{1+\frac{b}{x}+\frac{2020}{x^2}}\right)\right)=-4+\lim_{x\to\infty}\frac{x\left(1-1-\frac{b}{x}-\frac{2020}{x^2}\right)}{1+\sqrt{1+\frac{b}{x}+\frac{2020}{x^2}}}=-4+\frac{-b}{2}=1$$ ***2p***

Finalizare $a=1$ si $b=-10$ ***1p***

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