Barem de corectare

<table>
<tr><td rowspan="2">1.</td><td>

$1\epsilon G \Rightarrow \sqrt{1+2} = \sqrt{3} \epsilon\, G \Leftrightarrow \sqrt{0+3}\, \epsilon G \Rightarrow 0+4 = 4 \epsilon\, G$
Pres că $3k+1 \epsilon\, G \Rightarrow \sqrt{3k+1+2} = \sqrt{3k+3}\, \epsilon\, G \Rightarrow 3(k+1)+1 \epsilon\, G \Rightarrow 2014 = 3\cdot 671 + 1 \epsilon\, G$
Pe de altă parte
$4 \epsilon\, G \Rightarrow \sqrt{4+2} = \sqrt{6}\, \epsilon\, G \Leftrightarrow \sqrt{3+3}\, \epsilon\, G \Rightarrow 3+4 = 7 \epsilon\, G$
$7 \epsilon\, G \Rightarrow \sqrt{7+2} = \sqrt{9} = 3 \epsilon\, G$
$3 \epsilon\, G \Rightarrow \sqrt{3+2} = \sqrt{5}\, \epsilon\, G \Leftrightarrow \sqrt{2+3}\, \epsilon\, G \Rightarrow 2+4 = 6 \epsilon\, G$
Pres că $3k \epsilon\, G \Rightarrow \sqrt{3k+2}\, \epsilon\, G \Leftrightarrow \sqrt{3k-1+3}\, \epsilon\, G \Rightarrow 3k-1+4 = 3(k+1) \epsilon\, G$
$2013 = 3\cdot 671 \epsilon\, G \Rightarrow \sqrt{2013+2} = \sqrt{2015}\, \epsilon\, G$
G.M. 12/2014 – adaptare

</td><td>1 p
1 p
3 p
2 p</td></tr>
<tr><td>**TOTAL Subiectul 1**</td><td>7 p</td></tr>
<tr><td>2. a)</td><td>

$a \le \frac{a^2+1}{2};\ bc \le \frac{b^2+c^2}{2};\ \forall a,b,c \in \mathbb{R}_+^*$ Prin adunarea inegalităților se obține relația de la a)

</td><td>2 p</td></tr>
<tr><td rowspan="2">2. b)</td><td>

Analog
$\frac{3(b^2+c^2)+2}{3(a+bc)} \ge \frac{6(b^2+c^2)+4}{3(a^2+b^2+c^2+1)};\quad \frac{3(c^2+a^2)+2}{3(b+ac)} \ge \frac{6(c^2+a^2)+4}{3(a^2+b^2+c^2+1)}$
Adunând relațiile se obține:
$\frac{3(a^2+b^2)+2}{3(c+ab)} + \frac{3(b^2+c^2)+2}{3(a+bc)} + \frac{3(c^2+a^2)+2}{3(b+ac)} \ge \frac{6(a^2+b^2)+4}{3(a^2+b^2+c^2+1)} + \frac{6(b^2+c^2)+4}{3(a^2+b^2+c^2+1)} + \frac{6(c^2+a^2)+4}{3(a^2+b^2+c^2+1)} =$
$= \frac{12(a^2+b^2+c^2)+12}{3(a^2+b^2+c^2+1)} = \frac{12(a^2+b^2+c^2+1)}{3(a^2+b^2+c^2+1)} = 4$

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4 p</td></tr>
<tr><td>**TOTAL Subiectul 2**</td><td>7 p</td></tr>
<tr><td rowspan="2">3.</td><td>

Pentru $n=1 \Rightarrow \frac{1^2}{a_2+1} = \frac{2}{4} \Rightarrow a_2 = 3$
Pentru $n=2 \Rightarrow \frac{1^2}{a_1\cdot a_2+1} + \frac{2^2}{a_2\cdot a_3+1} = \frac{a_2+1}{8} \Leftrightarrow \frac{1}{4} + \frac{4}{3a_3+1} = \frac{4}{8} \Leftrightarrow \frac{4}{3a_3+1} = \frac{2}{8} \Leftrightarrow 3a_3+1 = 16$
$\Rightarrow a_3 = 5$
Presupunem că $a_n = 2n-1;\ a_{n+1} = 2n+1$ și demonstrăm prin inducție.
$\Rightarrow \frac{a_{n+1}}{8} + \frac{(n+1)^2}{a_{n+1}\cdot a_{n+2}+1} = \frac{a_{n+1}+1}{8}$
$\frac{2n-1+1}{8} + \frac{(n+1)^2}{(2n+1)\cdot a_{n+2}+1} = \frac{2n+2}{8}$
$\frac{(n+1)^2}{(2n+1)\cdot a_{n+2}+1} = \frac{1}{4} \Leftrightarrow (2n+1)a_{n+2}+1 = 4n^2+8n+4 \Leftrightarrow (2n+1)a_{n+2} = 4n^2+8n+3 \Leftrightarrow$
$(2n+1)a_{n+2} = (2n+1)(2n+3)$ $a_{n+2} = 2n+3$ Adev.

</td><td>1 p
1 p
3 p
2p</td></tr>
<tr><td>**TOTAL Subiectul 3**</td><td>7 p</td></tr>
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4.	Notăm R mijlocului medianei AD. Notăm $\frac{AM}{MB} = x$, $\frac{AN}{NC} = y$ $\Rightarrow \frac{AM}{AB} = \frac{x}{x+1} \Rightarrow \overrightarrow{AM} = \frac{x}{x+1} \cdot \overrightarrow{AB}$	**1 p**
	$\frac{AN}{AC} = \frac{y}{y+1} \Rightarrow \overrightarrow{AN} = \frac{y}{y+1} \cdot \overrightarrow{AC}$	**1 p**
	$\Rightarrow \overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN} = -\frac{x}{x+1}\overrightarrow{AB} + \frac{y}{y+1} \cdot \overrightarrow{AC} \quad (1)$	**1 p**
	$\overrightarrow{AR} = \frac{1}{2}\overrightarrow{AD} = \frac{1}{4}\left(\overrightarrow{AB} + \overrightarrow{AC}\right) = \frac{1}{4}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AC}$	**1 p**
	$\overrightarrow{MR} = \overrightarrow{MA} + \overrightarrow{AR} = -\frac{x}{x+1}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AC} = \frac{-4x + x + 1}{4(x+1)}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AC}$	**1 p**
	$\overrightarrow{MR} = \frac{1-3x}{4(x+1)}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AC} \quad (2)$	**1 p**
	$\overrightarrow{MR}$ și $\overrightarrow{MN}$ sunt coliniari $\overset{(1),(2)}{\Longleftrightarrow} \frac{-\frac{x}{x+1}}{\frac{1-3x}{4(x+1)}} = \frac{\frac{y}{y+1}}{\frac{1}{4}} \Leftrightarrow \frac{-x}{1-3x} = \frac{y}{y+1} \Leftrightarrow -xy - x = y - 3xy \Leftrightarrow 2xy = x + y \Leftrightarrow 2P = S$	**1 p**
	TOTAL Subiectul 4	**7 p**

Subiecte propuse de prof. BUD ADRIAN – Liceul Teoretic Negreşti Oaş