

BAREM CLASA XI

SUBIECTUL I

a) $A^2 = 5A$ (1p)

$$A^n = 5^{n-1} \cdot A \text{ (0,5p)}$$

Finalizare prin inductie (0,5p)

b) $\det X = 0$ (1p)

$$X^2 = t \cdot X, t \in \mathbb{R}, t = \text{Tr}(X) \text{ (1p)}$$

$$X^{2015} = t^{2014} \cdot X \Rightarrow X = \frac{1}{t^{2014}} \cdot A \text{ (1p)}$$

$$X^{2015} = \frac{1}{t^{2014 \cdot 2015}} \cdot 5^{2014} \cdot A \Rightarrow t^{2014} = \frac{1}{\sqrt[2015]{5^{2014}}} \text{ (1p)}$$

$$X = \frac{1}{\sqrt[2015]{5^{2014}}} \cdot A \text{ (1p)}$$

SUBIECTUL II

a) $\lim_{n \rightarrow \infty} ((\sqrt[3]{n^3 + 9n^2 + 1} - n) + (\sqrt[3]{n^3 - 9n^2 + 1} - n))$ (0,5p)

$$\lim_{n \rightarrow \infty} (\sqrt[3]{n^3 + 9n^2 + 1} - n) = 3 \text{ (0,5p)}$$

$$\lim_{n \rightarrow \infty} (\sqrt[3]{n^3 - 9n^2 + 1} - n) = -3 \text{ (0,5)}$$

Finalizare(0,5p)

b) $\lim_{n \rightarrow \infty} n((\sqrt[3]{n^3 + 9n^2 + 1} - (n+3)) + (\sqrt[3]{n^3 - 9n^2 + 1} - (n-3)))$ (2p)

$$\lim_{n \rightarrow \infty} n(\sqrt[3]{n^3 + 9n^2 + 1} - (n+3)) = -9 \text{ (1p)}$$

$$\lim_{n \rightarrow \infty} n(\sqrt[3]{n^3 - 9n^2 + 1} - (n-3)) = -9 \text{ (1p)}$$

Finalizare(1p)

SUBIECTUL III

a) $\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 3 & 2 \end{pmatrix}$ (1p)

$$\begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & \frac{3}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}^2 \text{ (1p); } \begin{pmatrix} 0 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \frac{3}{\sqrt{2}} & \sqrt{2} \end{pmatrix}^2 \text{ (1p)}$$

$$A = \begin{pmatrix} \sqrt{2} & \frac{3}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ \frac{3}{\sqrt{2}} & \sqrt{2} \end{pmatrix}$$

b) Presupunem, prin reducere la absurd, ca $AB = BA$ (1p)

$$\det(A^2 + B^2) = \det((A + iB)(A - iB)) = |\det(A + iB)|^2 \quad (2p)$$

$$\det \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} < 0 \quad (1p).$$

SUBIECTUL IV

$$x_1 = \cos \frac{\pi}{4} \quad (1p); \quad x_2 = \cos \frac{\pi}{8} \quad (1p); \quad x_n = \cos \frac{\pi}{2^{n+1}} \quad (1p); \quad x_1 \cdot x_2 \cdot \dots \cdot x_n = \frac{1}{2^n \sin \frac{\pi}{2^{n+1}}} \quad (2p);$$

$$\lim_{n \rightarrow \infty} x_1 \cdot x_2 \cdot \dots \cdot x_n = \frac{2}{\pi} \quad (2p).$$