Concursul Naţional de Matematică Aplicată " Adolf Haimovici ",
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filiera teoretică: profil real: Ştiinţe ale naturii
cl. a X-a
Barem

1.a) $a^3 = 1+48\sqrt[3]{65}-12\sqrt[3]{65^2}$ 1 punct
$b^3 = 1+36\sqrt[3]{147}-48\sqrt[3]{63}$ 1 punct
b) $A = \sqrt[3]{65}$ 2 puncte
$B = 4$ 2 puncte
$B = 4 = \sqrt[3]{64} < \sqrt[3]{65} = A$ 1 punct

2.a) $\alpha \in \mathbf{C}\setminus\mathbf{R}$ şi $\alpha^3 = 1 \Rightarrow \alpha \neq 1$ şi $\alpha^3 - 1 = 0 \Rightarrow (\alpha-1)(\alpha^2+\alpha+1)=0$ 1 punct
$\Rightarrow \alpha^2+\alpha+1=0$. Atunci $\alpha^n + \alpha^{n+1} + \alpha^{n+2} = \alpha^n(\alpha^2+\alpha+1) = 0$
unde $n \in \mathbf{N}$ 1 punct
b) $2013 = 3\cdot 671$ iar $\alpha^n + \alpha^{n+1} + \alpha^{n+2} = 0$ unde $n\in\mathbf{N} \Rightarrow$ 1 punct
$1 + \alpha + \alpha^2 + ... + \alpha^{2013} = 1 + 671\cdot 0 = 1$ 1 punct
c) $\alpha^3 = 1 \Rightarrow A = \{1, \alpha, \alpha^2\}$ 1 punct
din a) şi b) $\Rightarrow B = \{1, 1+\alpha, 0\}$ 1 punct
deci $A \cap B = \{1\}$ 1 punct

3. $z\cdot\overline{z} = |z|^2$ 1 punct
$|z_1+z_2|^2 + |z_1-z_2|^2 = (z_1+z_2)(\overline{z_1}+\overline{z_2}) + (z_1-z_2)(\overline{z_1}-\overline{z_2}) = 2(|z_1|^2+|z_2|^2)$ 3 puncte
Interpretare geometrică:
OACB paralelogram (unde $O(0,0), A(z_1), C(z_1+z_2), B(z_2)$: $z_O+z_C=z_A+z_B$) 1punct
$OA = |z_1| = BC$, $OB = |z_2| = AC$ 1 punct
$OC = |z_1+z_2|$, $AB = |z_1-z_2|$ 1 punct

4.Media aritmetică $\geq$ Media geometrică (3 numere) 1 punct
$$\left(\frac{a^2}{bc}\right)^{\lg\frac{b}{c}} + \left(\frac{b^2}{ac}\right)^{\lg\frac{c}{a}} + \left(\frac{c^2}{ab}\right)^{\lg\frac{a}{b}} \geq 3\sqrt[3]{\left(\frac{a^2}{bc}\right)^{\lg\frac{b}{c}}\left(\frac{b^2}{ac}\right)^{\lg\frac{c}{a}}\left(\frac{c^2}{ab}\right)^{\lg\frac{a}{b}}}$$ 2 puncte
$$\left(\frac{a^2}{bc}\right)^{\lg\frac{b}{c}}\left(\frac{b^2}{ac}\right)^{\lg\frac{c}{a}}\left(\frac{c^2}{ab}\right)^{\lg\frac{a}{b}} = 1$$ 2 puncte
Egalitate pentru a = b = c 1 punct
Finalizare 1 punct