OLIMPIADA DE MATEMATICĂ
Etapa locală – Constanța, 16.02.2013
Clasa a VII-a
Barem de corectare și notare

1. a.	$u(p^2) \in \{0, 1, 4, 5, 6, 9\}$, $u(5n + 2) \in \{2, 7\}$, finalizare	3p
1. b.	$u(2^{2009}) = 2$, $u(2^{2008}) = 6$, $u(2^{2007}) = 8$, $u(2^{2006}) = 4$	2p
	$u(x^2) = u(2 + 6 + 8 = 4 + 7) = 7$, finalizare $x \in \mathbf{R} \setminus \mathbf{Q}$.	2p
2. a)	$(1 + 2 + ... + 2012)^2 = (1006 \cdot 2013)^2$	1p
	$\sqrt{2013^2(1006^2+2013)} = 2013\sqrt{1006^2+2013} = 2013 \cdot 1007$	2p
	$1 + 2 + 3 + ... + 2013 = \frac{2013 \cdot 2014}{2} = 2013 \cdot 1007$	1p
2. b)	$\frac{3a+1}{3a-5} = \frac{3a-5+6}{3a-5} = 1 + \frac{6}{3a-5} \in Z \Rightarrow 3a-5 \in D_6, a \in N \Rightarrow$ $a = 1$ *sau* $a = 2$, deci $A = \{-2 ; 7\}$.	1p
	$\frac{4b+1}{4b-1} = \frac{4b-1+2}{4b-1} = 1 + \frac{2}{4b-1} \in Z \Rightarrow 4b-1 \in D_2, b \in N \Rightarrow$ $b = 0$ deci $B = \{-1\}$	1p
	Finalizare, $A \cap B = \phi$	1p
3.	Justificarea $CD \parallel MN$, $MN = CD = 2 \Rightarrow MNCD$ paralelogram, $MC \cap ND = \{O\} \Rightarrow \frac{DO}{NO} = \frac{MO}{OC} = 1$	2p
	G centru greutate $\Delta CDM \Rightarrow \frac{GO}{GD} = \frac{1}{2} \Rightarrow \frac{GN}{ND} = \frac{2}{3}$	1p
	$MN \cap AD = \{E\}$, $CD \parallel MN$, $MN = CD = 2 \Rightarrow \frac{MN}{EN} = \frac{2}{3}$	2p
	In ΔNED, $\frac{GN}{ND} = \frac{MN}{EN} \Rightarrow MG \parallel DE$, $m(\widehat{GMN}) = 90^0 \Rightarrow$	1p
	$GM \perp NE$, $MG \parallel AD \Rightarrow m(\measuredangle E) = m(\measuredangle A) = 90^0 \Rightarrow$ *ABCD* trapez dreptunghic.	1p
4.a.	Aplicarea teoremei bisectoarei în ΔABD, $AD = 3$, $AB = 9 \Rightarrow$ $DE = EO = \frac{1}{4}DB$ și în ΔALD, $AD = DL = 3$ *si* $LC = 6$.	1p
	ABCD paralelogram, $\frac{LC}{DC} = \frac{CO}{CM} = \frac{2}{3} \Rightarrow LO \parallel DN$, $DN \perp AP \Rightarrow LO \perp AP$ $\frac{DM}{NM} = \frac{AD}{NC} = \frac{1}{3} \Rightarrow \frac{DM}{DN} = \frac{1}{4}$ și $\frac{LO}{DM} = \frac{2}{3} \Rightarrow \frac{LO}{DN} = \frac{1}{6}$	2p
4.b.	Justificarea $\frac{AE}{EP} = \frac{DE}{BE} = \frac{AD}{BP} = \frac{1}{3}$ și $\frac{AD}{NC} = \frac{1}{3} \Rightarrow \frac{AD}{PN} = \frac{1}{5}$	1p
	$\frac{EO}{DO} = \frac{MO}{AO} = \frac{1}{2} \Rightarrow ME \parallel AD \parallel BC$, $\frac{AD}{PN} = \frac{1}{5}$ *si* $\frac{EM}{AD} = \frac{1}{2} \Rightarrow \frac{EM}{PN} = \frac{1}{10}$	1p
4.c.	$\frac{AD}{CN} = \frac{1}{3}$, $AD \parallel CN \Rightarrow (\exists)K, AN \cap DC = \{K\}, \frac{KD}{CD} = \frac{1}{2}, D \in (CK)$.	1p
	$\frac{RD}{BN} = \frac{1}{3}$, $RD \parallel CP \Rightarrow (\exists)K', PR \cap DC = \{K'\}, \frac{K'D}{CD} = \frac{1}{2}, D \in (CK')$, Deci $K' = K$ și *AN*, *CD* și *PR* sunt drepte concurente.	1p

Figura problemei 7.4.

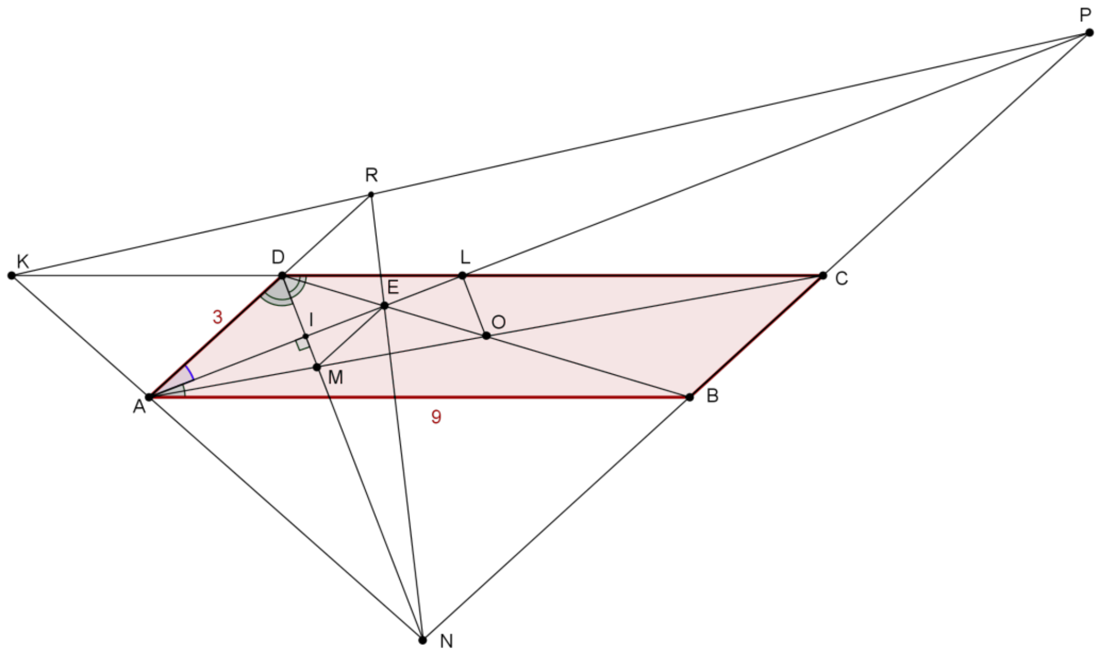

Figura problemei 7.3.