CONCURSUL DE MATEMATICĂ APLICATĂ„ADOLF HAIMOVICI”

Etapa locală – Constanța, 16.02.2013

filiera teoretică: profil real, specializarea ştiinţe ale naturii

Clasa a X-a

Barem de corectare și notare

Subiectul 1

a) Pentru $x^3 = 4-3x$ ……………………………………………………..……**(1p)**

Deci $x^3+3x-4=0$ …………………………………………………………….**(1p)**

Pentru $x^3+3x-4=(x-1)(x^2+x+4)$…………………………………… **(1p)**

Deoarece $x\in R \Rightarrow x=1$…………………………………………………..**(1p)**

b) Pentru $10^k \le x < 10^{k+1} \Rightarrow [\lg x]=k, k\in N$ ………………………………...**(2p)**

Calculul sumei $S=90+900\cdot 2+1014\cdot 3$ ………………………………….……**(1p)**

Subiectul 2

a) Pentru $\operatorname{Im} f=(-\infty,m]\cup(1,\infty)$…………………………………….…...**(2p)**

Discutie pentru bijectivitate si aflarea lui $m=1$ ………………………………...**(2p)**

b) Inversa pentru fiecare ramura……………………………………………**(1p+1p)**

Finalizare………………………………………………………………….….**(1p)**

Subiectul 3

a) $\lg(x-3y)^2=\lg 4y^2$ ………………………………..…………………………**(1p)**

$(x-3y)^2=4y^2$ …………………………………………………………………**(1p)**

$$\begin{cases} x^2-6xy+5y^2=0 \\ \left(\frac{x}{y}\right)^2-6\left(\frac{x}{y}\right)+5=0 \end{cases}$$ ……………………………………………………………**(1p)**

$\frac{x}{y}=1$ nu convine; $\frac{x}{y}=5$ solutie………………………..…………………………**(1p)**

b) $a=\frac{\log_{12}3}{1+\log_{12}5}$ ……………………………………………………………...…**(1p)**

$b=\frac{\log_{12}5}{1+\log_{12}5}$ ……………………………………………………………...…**(1p)**

$$\frac{1-a-b}{2(1-b)}=\frac{1-\frac{\log_{12}3}{1+\log_{12}5}-\frac{\log_{12}5}{1+\log_{12}5}}{2\left(1-\frac{\log_{12}5}{1+\log_{12}5}\right)}=\frac{1-\log_{12}3}{2}=\frac{\log_{12}4}{2}=\log_{12}2 \Rightarrow 12^{\log_{12}2}=2^{\log_{12}12}=2$$ ……**(1p)**

Subiectul 4

a) $S=z_1^n+z_2^n, z_1=1+i, z_2=1-i$ ……………………...……………………**(1p)**

$\overline{S}=z_2^n+z_1^n \Rightarrow S=\overline{S}\Rightarrow S\in R$ …………………………………………………**(1p)**

$z_1^2=2i; z_2^2=-2i; z_1^{2013}=-2^{1006}\cdot z_1; z_2^{2013}=-2^{1006}\cdot z_2$ ………………………..……**(1p)**

$z_1^{2013}+z_2^{2013}=-2^{1006}\cdot(z_1+z_2)=-2^{1007}$, unde $z_1+z_2=2$ ………………..………**(1p)**

b) $z=0$ solutie

Daca $z\ne 0 \Rightarrow z^{2014}=\bar{z}\cdot z=r^2, r>0$ …………………………….………..**(1p)**

Rezulta $r^{2014}=r^2 \Rightarrow r=1$ ……………………………………………...…**(1p)**

$z^{2014}=1\Rightarrow z_k, k=\overline{0,2013}$ ……………………………………………...……**(1p)**