Barem de corectare OLM 2016 Clasa a XII-a

1. $x+x^{n+1}+x^{2n+1}+x^{3n+1}=x\left(1+x^n\right)\left(1+x^{2n}\right)$ ....
(1p)

$I=\int\frac{dx}{x\left(1+x^n\right)\left(1+x^{2n}\right)},\ J=\int\frac{x^n dx}{x\left(1+x^n\right)\left(1+x^{2n}\right)}$ .... **(1p)**

$I+J=\int\frac{x^{2n-1}}{x^{2n}\left(1+x^{2n}\right)}dx=\frac{1}{2n}\ln\frac{1+x^{2n}}{x^{2n}}+c_1,\ c_1\in R$ ....
(1p)

$\frac{1-x^n}{x\left(1+x^n\right)\left(1+x^{2n}\right)}=\frac{1}{x+x^{n+1}}-\frac{x^{n-1}}{1+x^{2n}}$ .... **(1p)**

$I-J=\int\frac{x^{n-1}}{x^n(1+x^n)}dx-\int\frac{x^{n-1}}{1+\left(x^n\right)^2}dx=\frac{1}{n}\ln\frac{1+x^n}{x^n}-\frac{1}{n}arctg\left(x^n\right)+c_2,\ c_2\in R$ .... **(2p)**

Finalizare $I=\frac{1}{4n}\ln\frac{\left(1+x^{2n}\right)\left(1+x^n\right)^2}{x^{4n}}-\frac{1}{2n}arctg\left(x^n\right)+c,\ c\in R$ .... **(1p)**

Sau

$I=\int\frac{x^{n-1}}{x^n\left(1+x^n\right)\left(1+x^{2n}\right)}dx,\ x^n=t\Rightarrow I_t=\frac{1}{n}\int\frac{dt}{t(1+t)\left(1+t^2\right)}$ .... **(3p)**

Rezolvarea integralei .... **(4p)**

2. $(xy)^{n+1}=x^{n+1}\cdot y^{n+1}\Leftrightarrow(xy)^n xy=x^{n+1}\cdot y^{n+1}\Leftrightarrow x^n y^n xy=x^{n+1}\cdot y^{n+1}\Leftrightarrow y^n x=xy^n$ ....
(2p)

$(xy)^{n+2}=x^{n+2}\cdot y^{n+2}\Leftrightarrow(xy)^{n+1}xy=x^{n+2}\cdot y^{n+2}\Leftrightarrow x^{n+1}y^{n+1}xy=x^{n+2}\cdot y^{n+2}\Leftrightarrow y^{n+1}x=xy^{n+1}$ ....
(2p)

$y^{n+1}x=y\cdot y^n x=y\cdot xy^n$ .... **(1p)**

$y\cdot xy^n=xy^{n+1}\Leftrightarrow yx=xy$ .... **(2p)**

3. *f* continuă $\Rightarrow$ *f* admite primitive pe R; fie $F:R\to R$ o primitivă pentru *f*. Din relaţia dată avem $F(bt)-F(at)\le F(b)-F(a),\forall t\in(0,2)$ .... **(1p)**

Notăm $F(bt)-F(at)=G(t),\ G:(0,2)\to R, G$ derivabilă $\Rightarrow G(t)\le G(1),\forall t\in(0,2)$ .... **(2p)**

$t=1$ punct de maxim pentru funcţia G .... **(1p)**

Din teorema lui Fermat rezultă că $G'(1)=0$ ....
(1p)

$G'(t)=F'(bt)\cdot b-F'(at)\cdot a=f(bt)\cdot b-f(at)\cdot a$ .... **(1p)**

$G'(1)=f(b)\cdot b-f(a)\cdot a=0\Leftrightarrow af(a)=bf(b)$ .... **(1p)**

4. a) $\forall\, x, y \in H$, $x * y \in H$ .... **(2p)**

$\forall\, x \in H, x' \in H$ ....

(2p)

b) Se ştie că *f* este bijectivă .... **(1p)**

f este morfism deoarece $f(x * y) = f(x^{\ln y}) = \ln\left(x^{\ln y}\right) = \ln x \cdot \ln y = f\left(x\right) \cdot f\left(y\right)$ .... **(2p)**